Overview of Complex Networks

Complex Networks, SFI Summer School, June, 2010

Prof. Peter Dodds

Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont

Outline

Something of a plan:

- Lecture 1: Overview; Background
- Lecture 2: Random, Scale-free, and Small-World networks
- Lecture 3: Models of Contagion
- Lecture 4: Transportation networks; Discovering structure

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- Three versions (all in pdf):
  1. Presentation,
  2. Flat Presentation,
  3. Handout (2x2).
- Presentation versions are navigable and hyperlinks are clickable.
- Web links look like this (⊞).
- References in slides link to full citation at end. [2]
- Citations contain links to papers in pdf (if available).
- 50 hours of lectures → 5 hours.
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Bonus materials:

Graduate Course Websites:

- SFI Summer School Course (this one!):

  http://www.uvm.edu/∼pdodds/teaching/courses/2010-06SFI-networks/ (⊞)
- Principles of Complex Systems (⊞), University of Vermont
- Complex Networks (⊞), University of Vermont

Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
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Bonus materials:

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [4]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  "The structure and function of complex networks" [15]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)

Basic definitions

Complex System—Some ingredients:

- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence—'More is Different' [2]

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Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

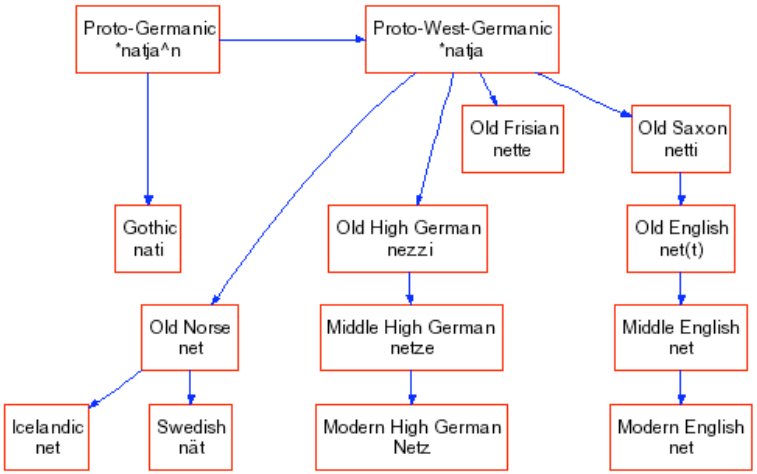
- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

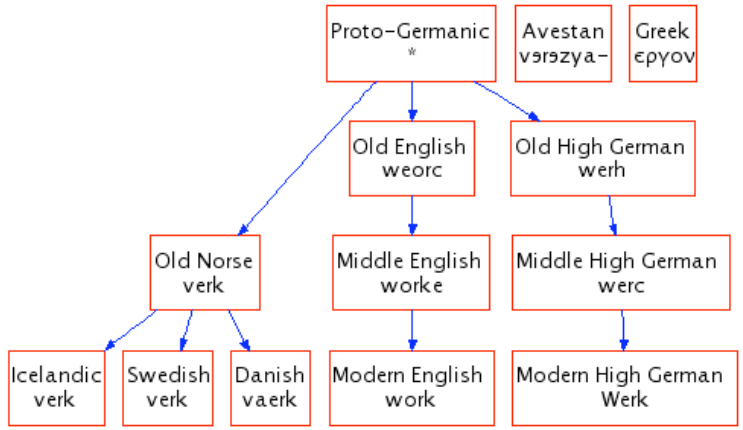
Ancestry:

Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appears to have long meant purposeful action.



The network of Germanic `net' words



The network of `work' words

- ‘Network’ = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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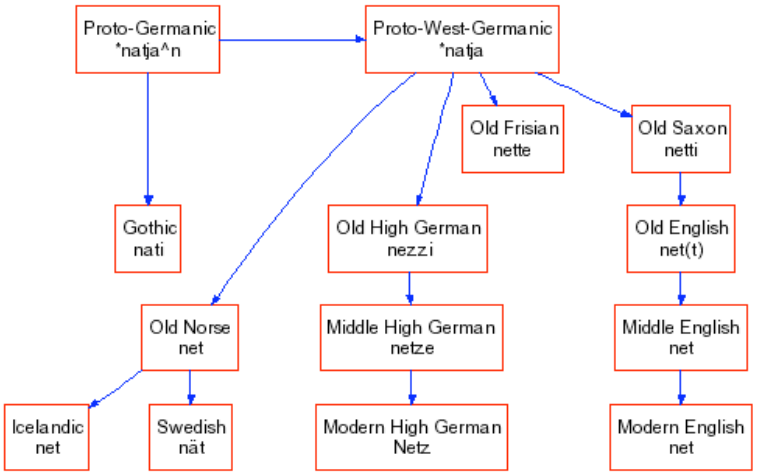

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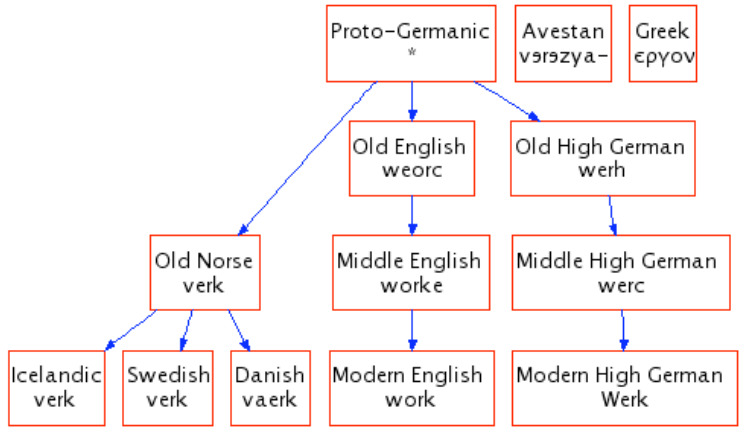

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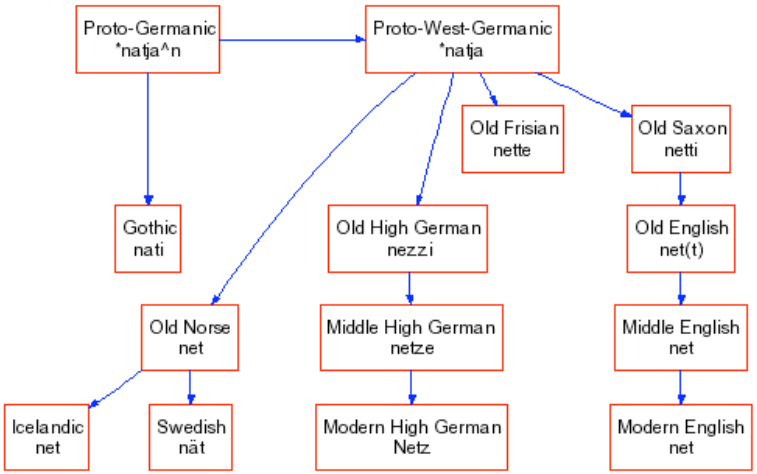

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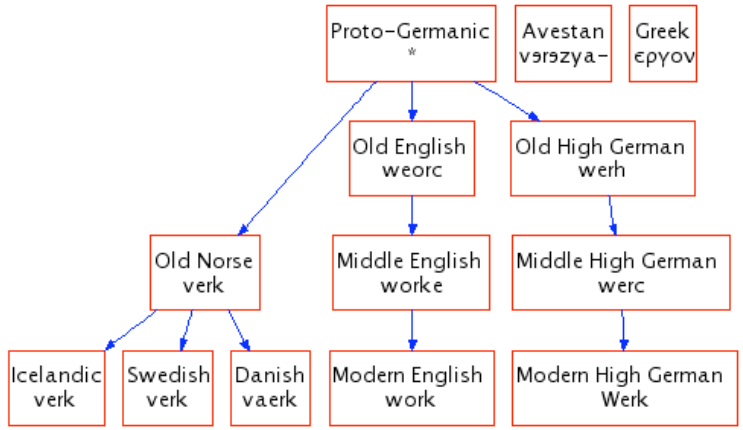

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First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

- Natural → man-made
- Physical connections → Wire-less connections → abstract connections

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Key Observation:

- Many complex systems
  can be viewed as complex networks
  of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:

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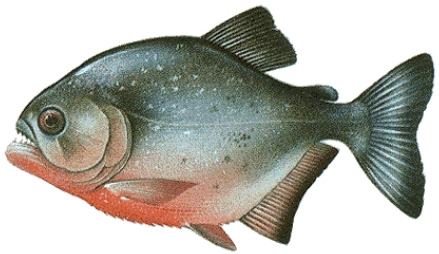

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- Hunt in packs.
- Feast on new and interesting ideas
  (see chaos, cellular automata, ...)

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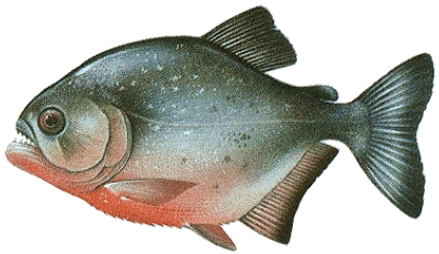

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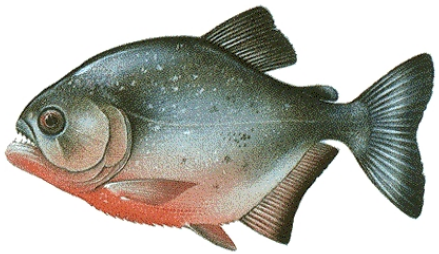

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Popularity (according to ISI Web of Knowledge)

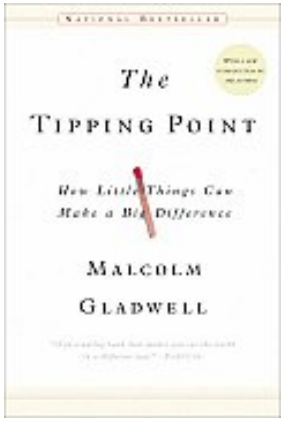
"Collective dynamics of 'small-world' networks" [21]

- Watts and Strogatz
  Nature, 1998
- Cited $\approx$ 4325 times (as of June 7, 2010)
- Over 1100 citations in 2008.

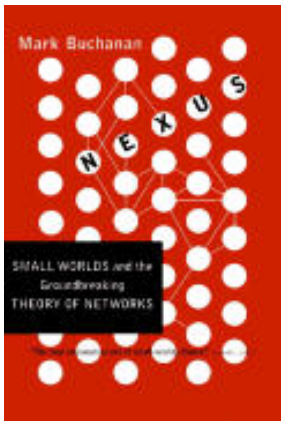
"Emergence of scaling in random networks" [3]

- Barabási and Albert
  Science, 1999
- Cited $\approx$ 4769 times (as of June 7, 2010)
- Over 1100 citations in 2008.

Popularity according to books:

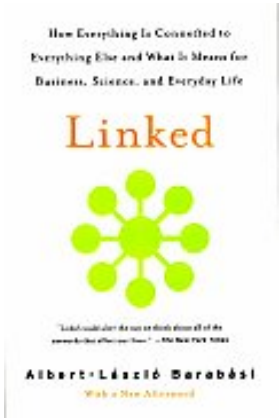


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [10]

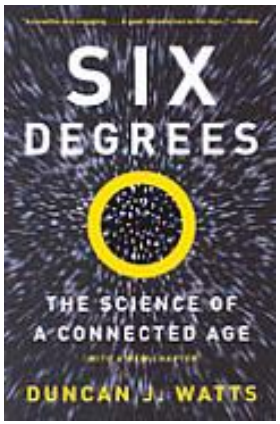


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [20]

Numerous others:

- Complex Social Networks—F. Vega-Redondo [19]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [16]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [6]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [9]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

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* *If this is upsetting, maybe string theory is for you...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (□)
- "The Unreasonable Effectiveness of Data," Halevy et al. [11].

But:

- For scientists, description is only part of the battle.
- We still need to understand.

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- "The Unreasonable Effectiveness of Data," Halevy et al. [11].

But:

- For scientists, description is only part of the battle.
- We still need to understand.

More observations

- Web-scale data sets can be overly exciting.

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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

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Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- $k_i = 0,1,2,\ldots$.
- Notation: the average degree of a network = $\langle k \rangle$
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

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Adjacency matrix:

- We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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Examples

So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
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- Neural networks
- Trees and leaves
- Blood networks
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- Road networks
- Power grids

- Distribution (branching) vs. redistribution (cyclical)

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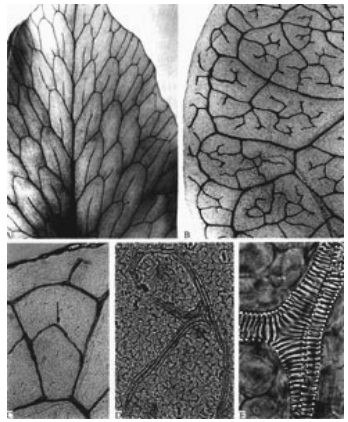

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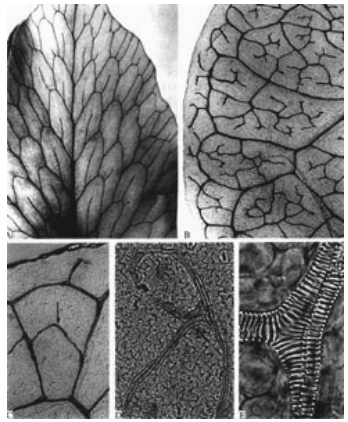

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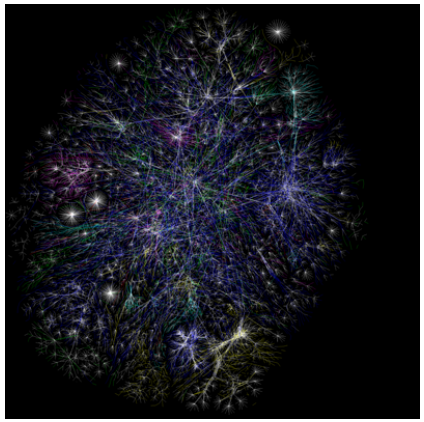

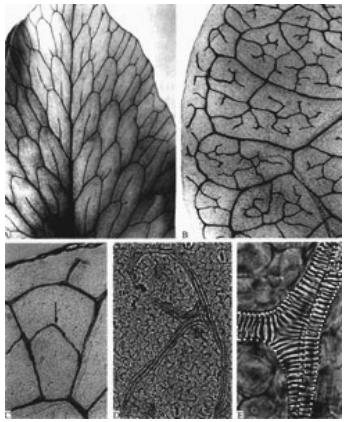

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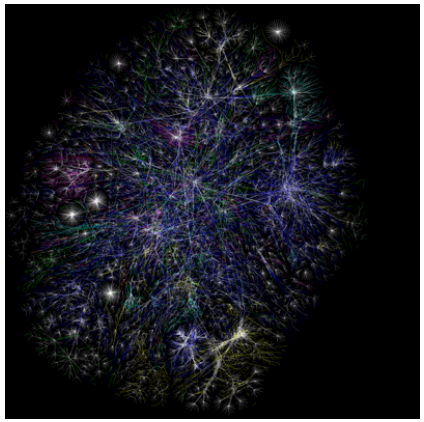

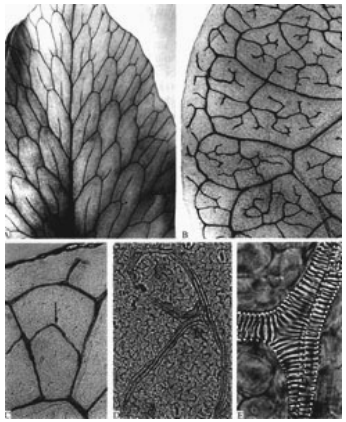

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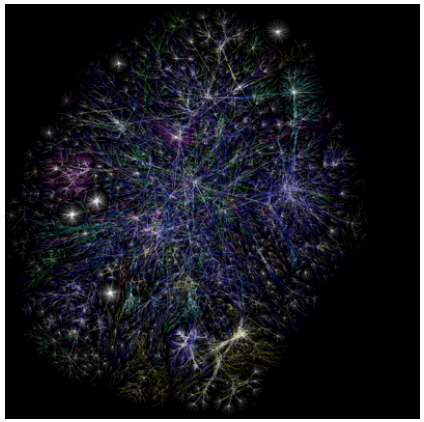

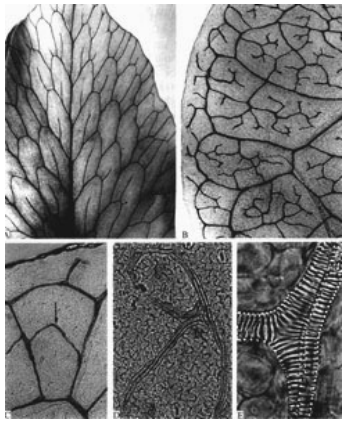

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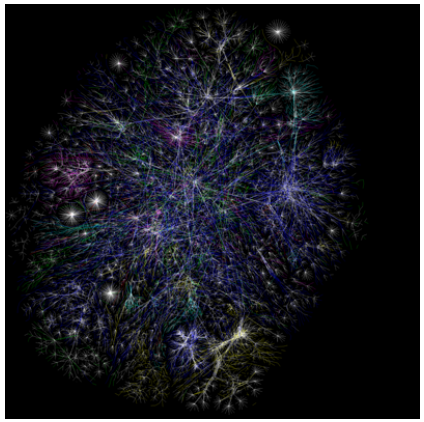

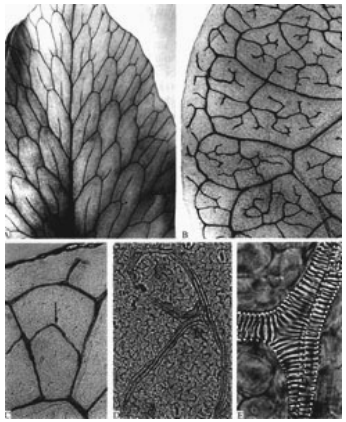

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- Food webs: who eats whom
- The World Wide Web (?)
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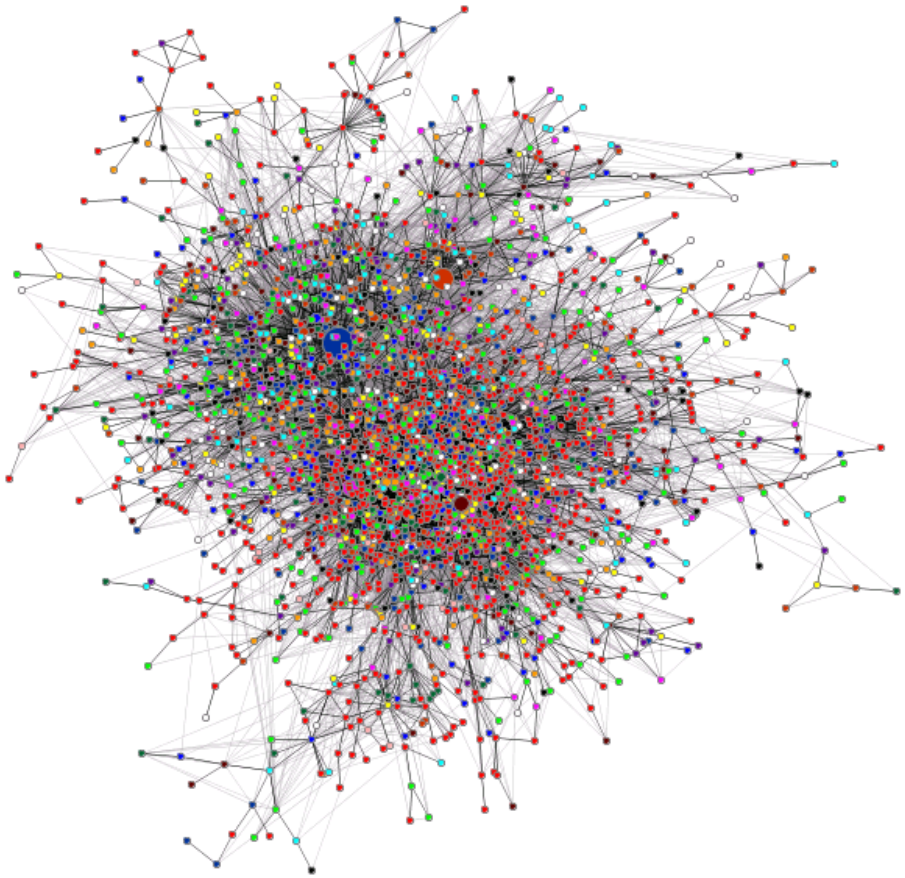

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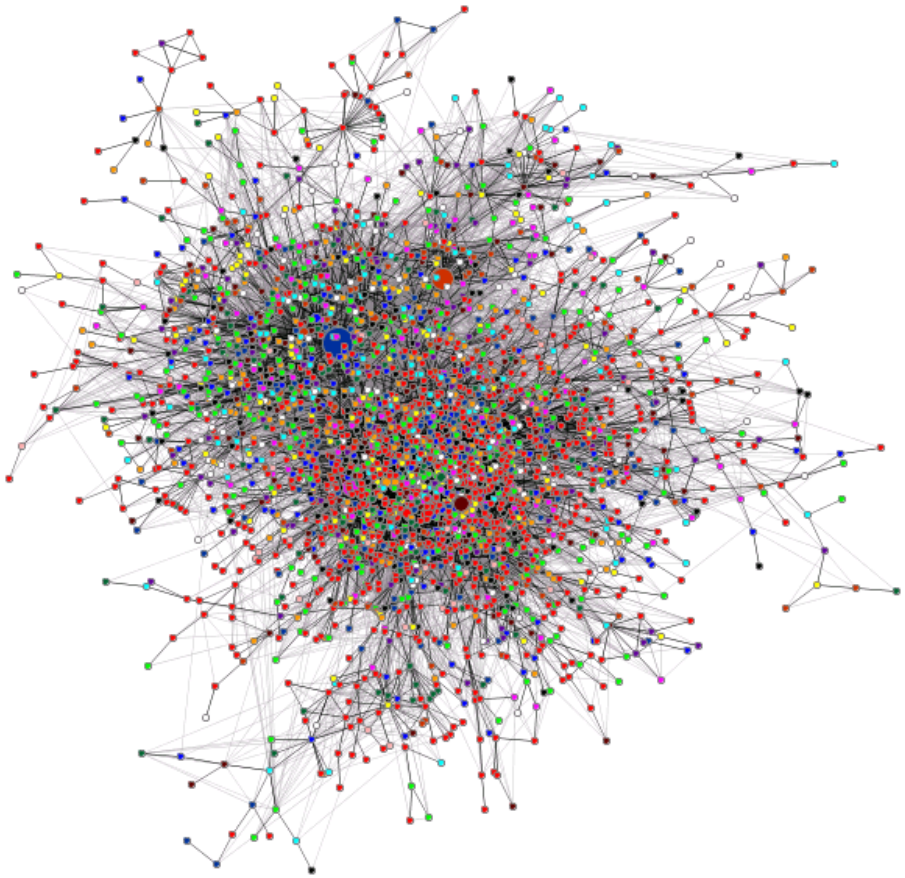

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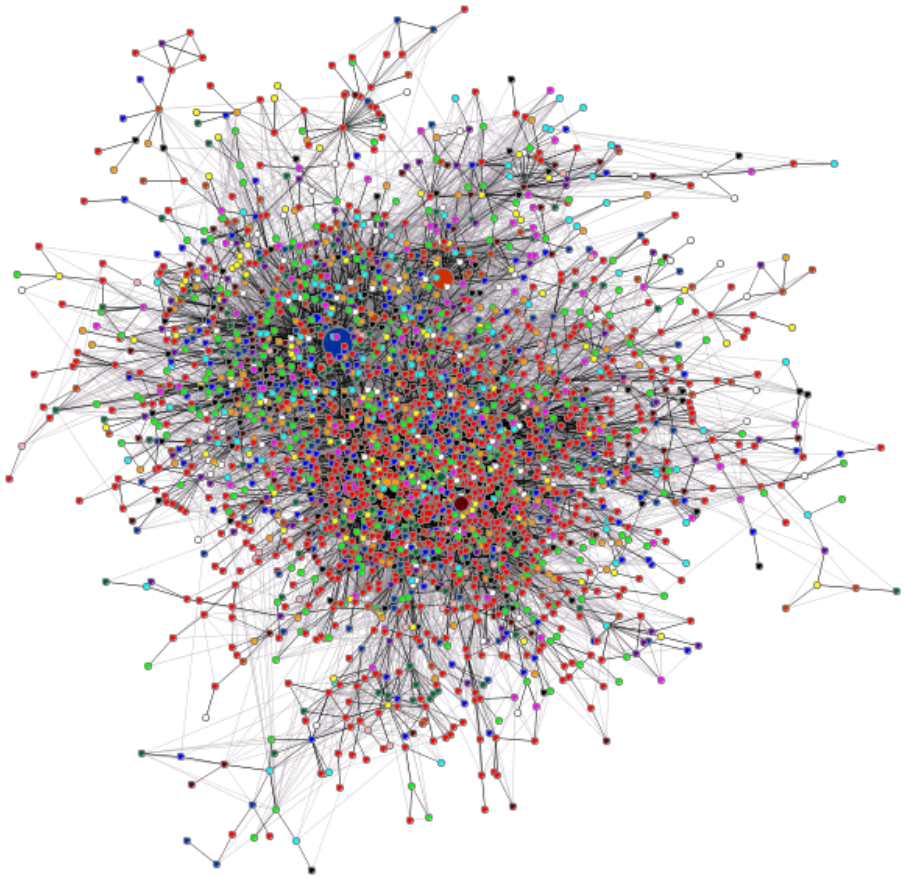

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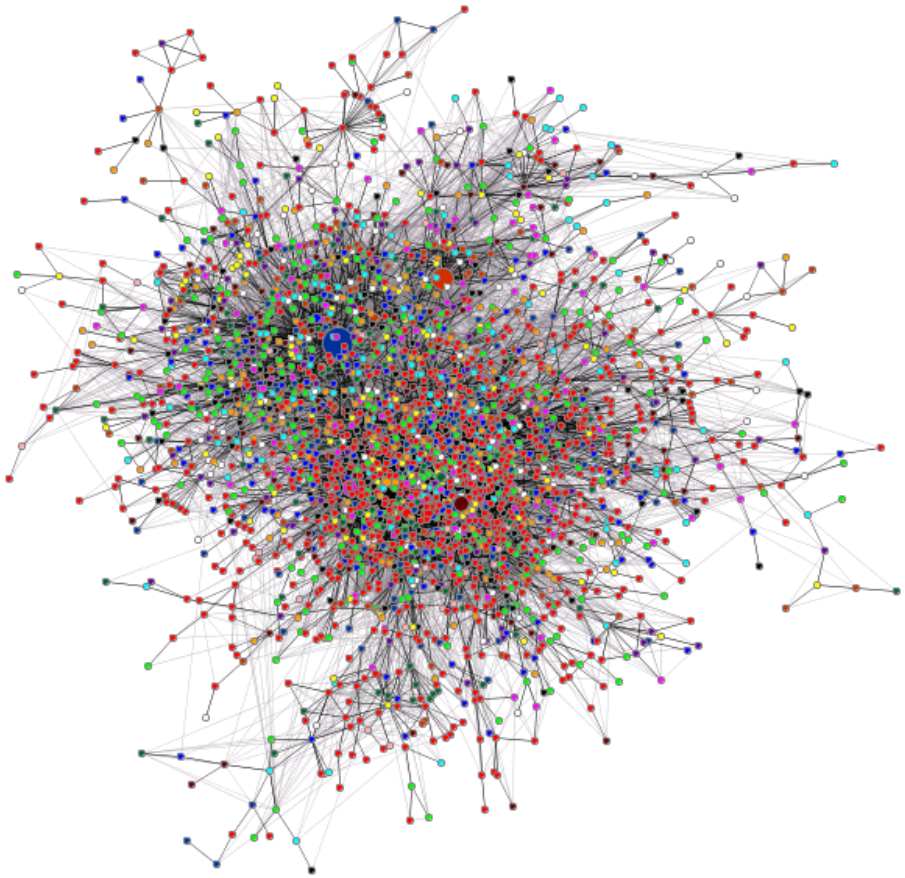

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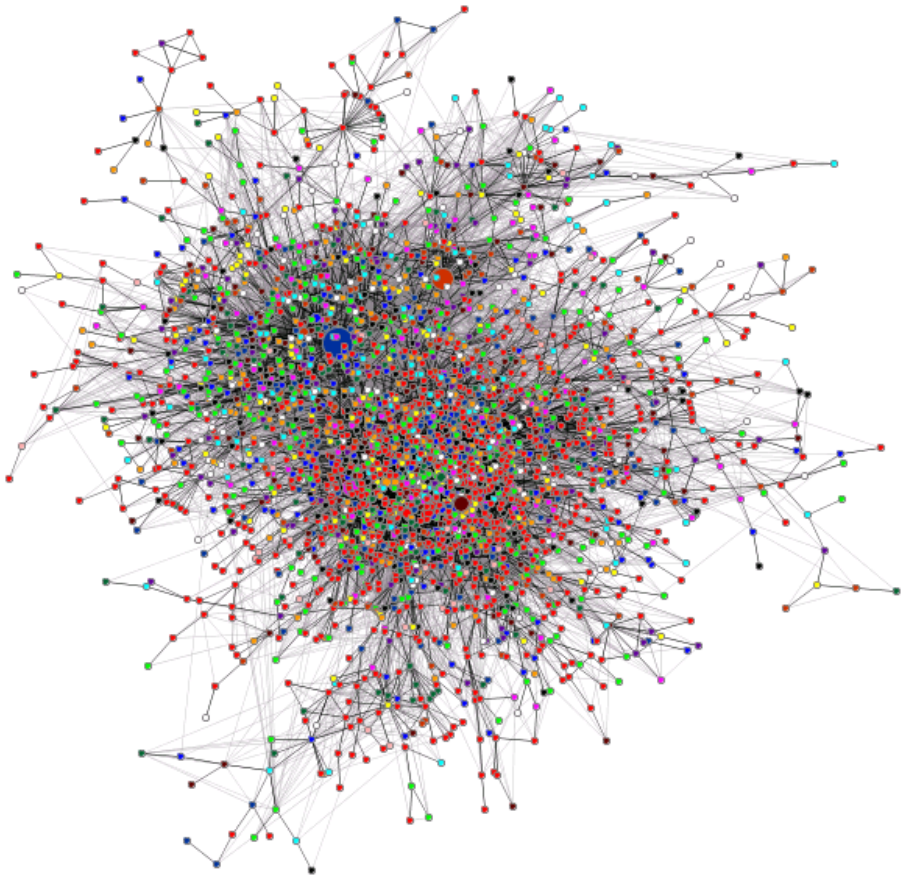

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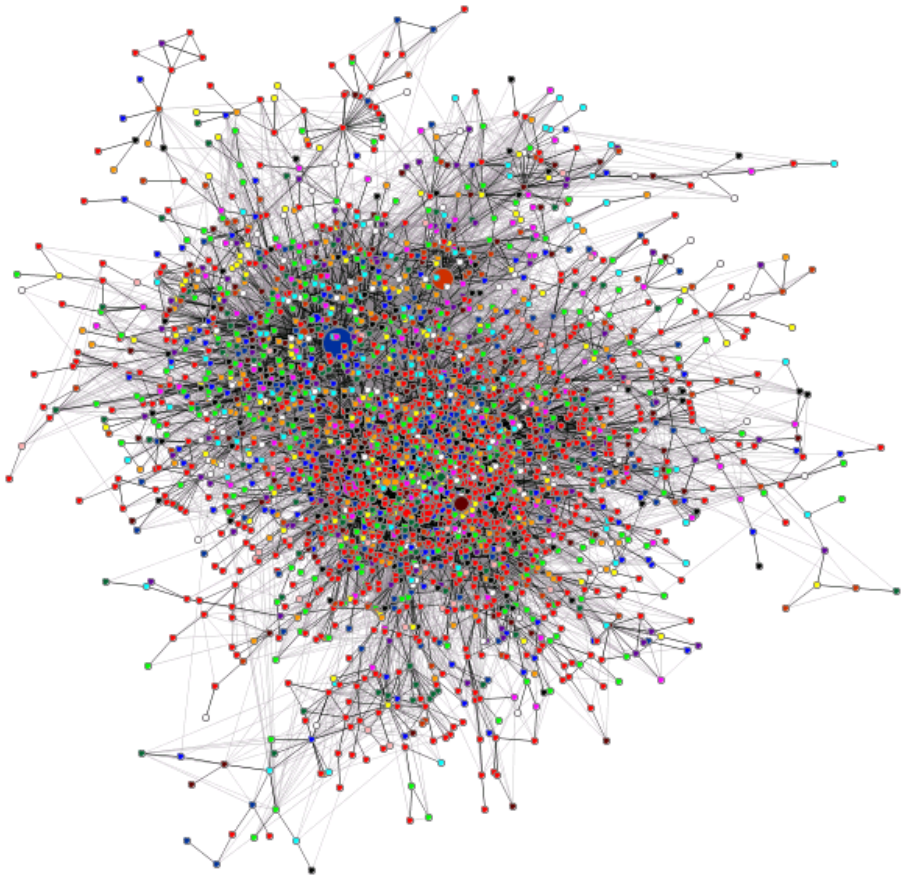

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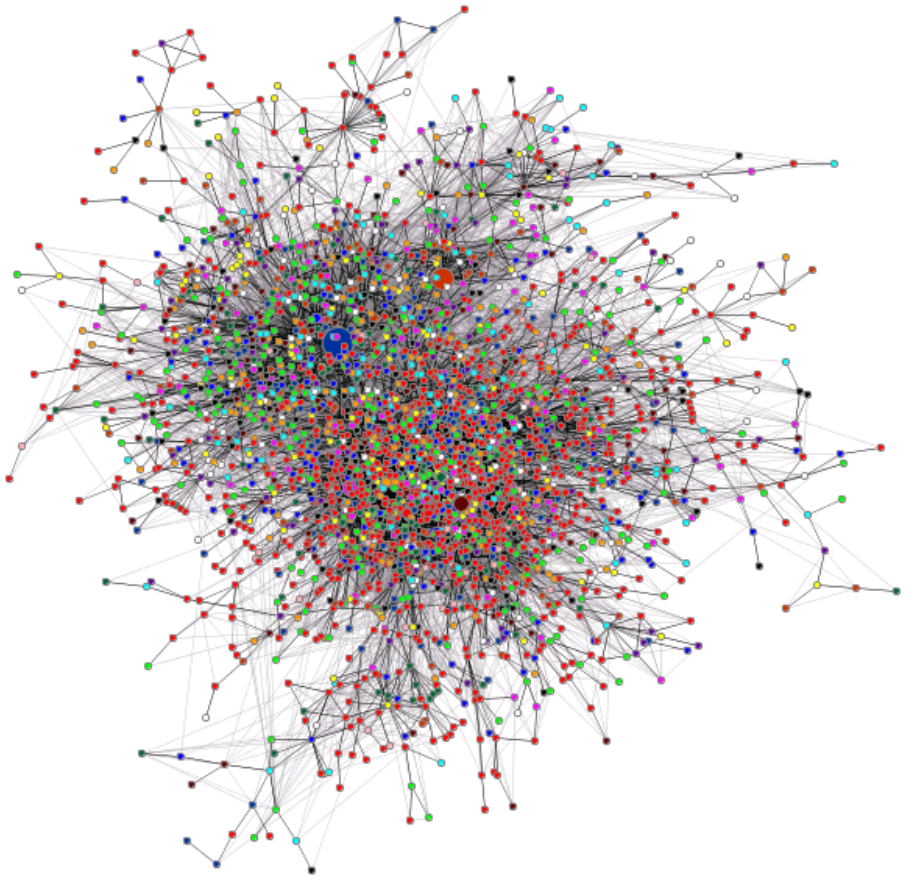

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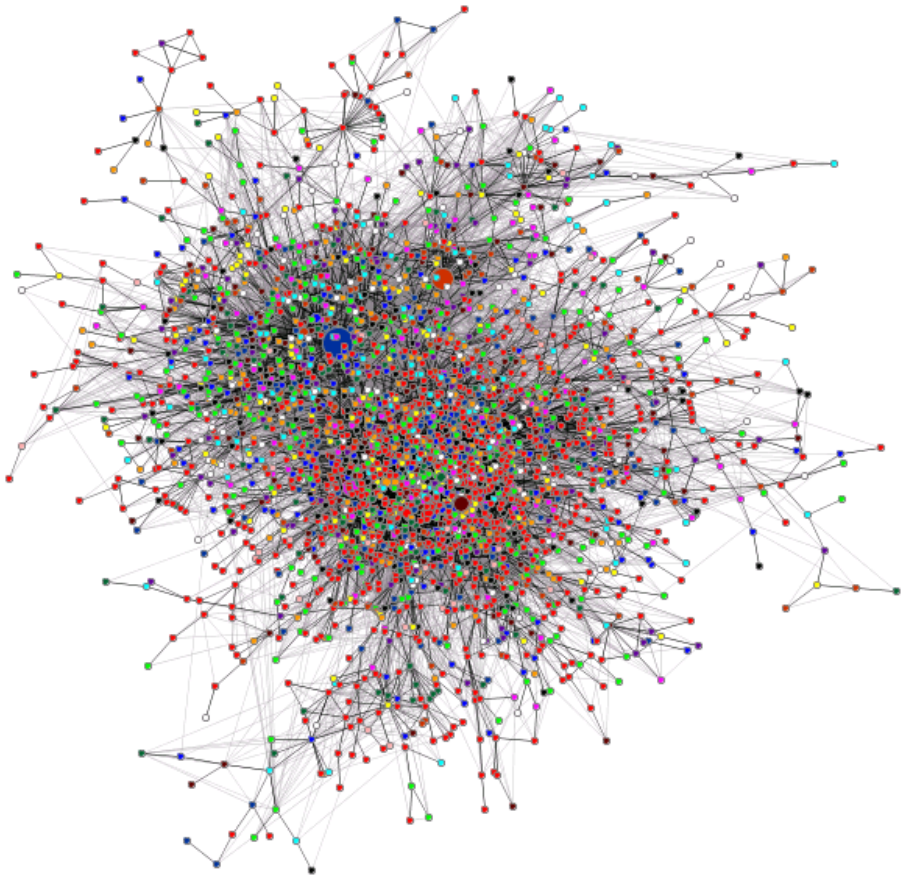

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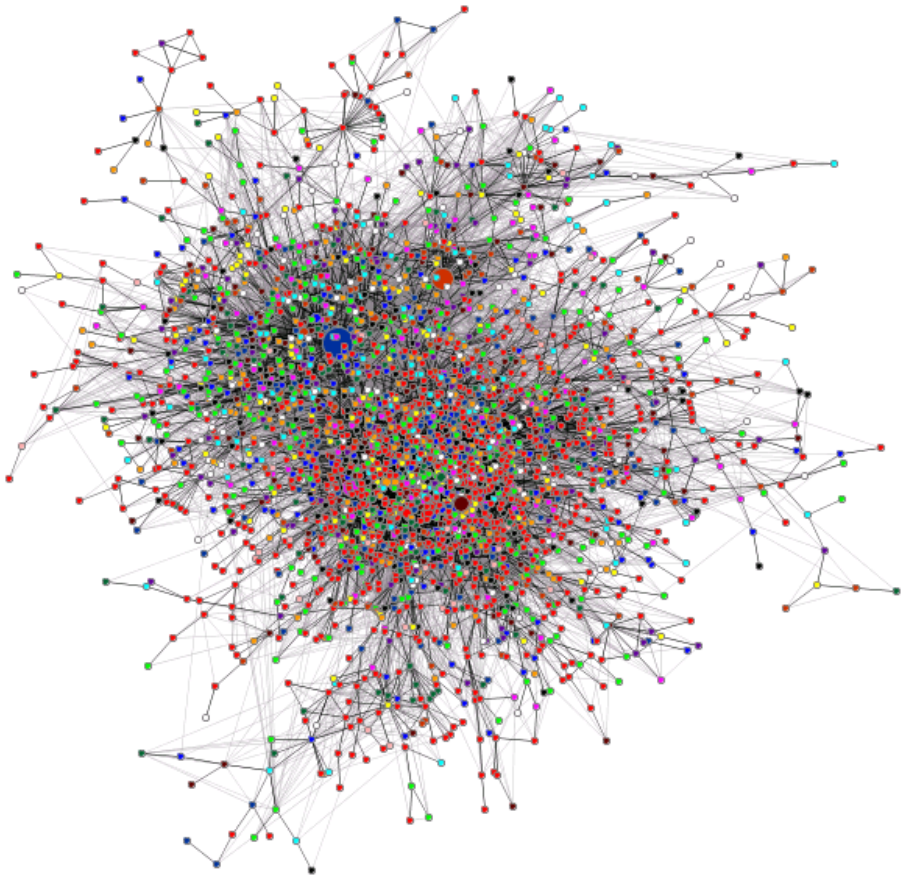

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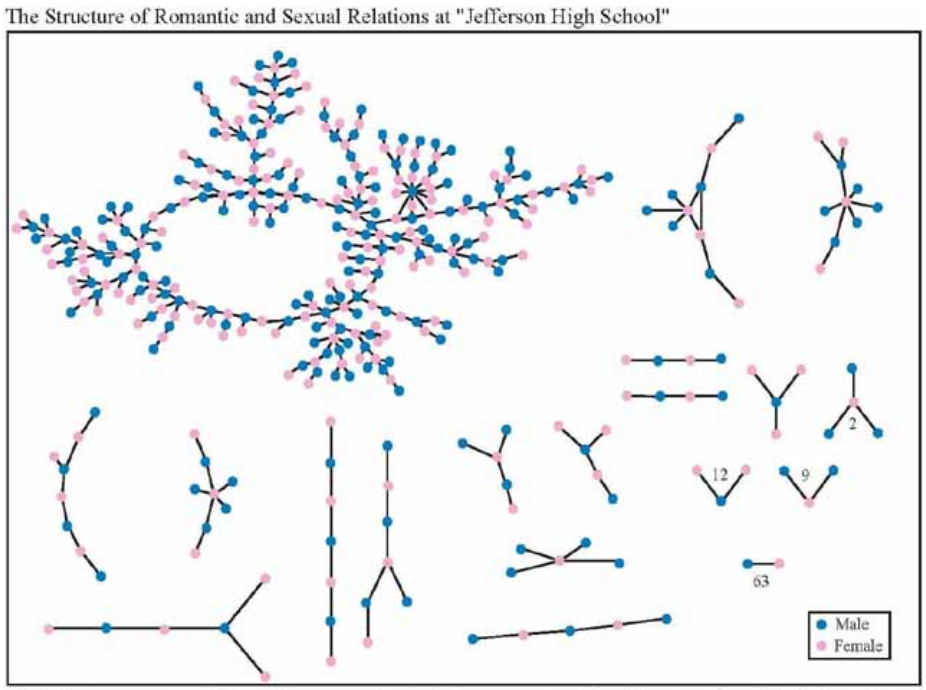
- Acquaintances
- Boards and directors
- Organizations
- facebook.com (⊞), twitter.com (⊞)



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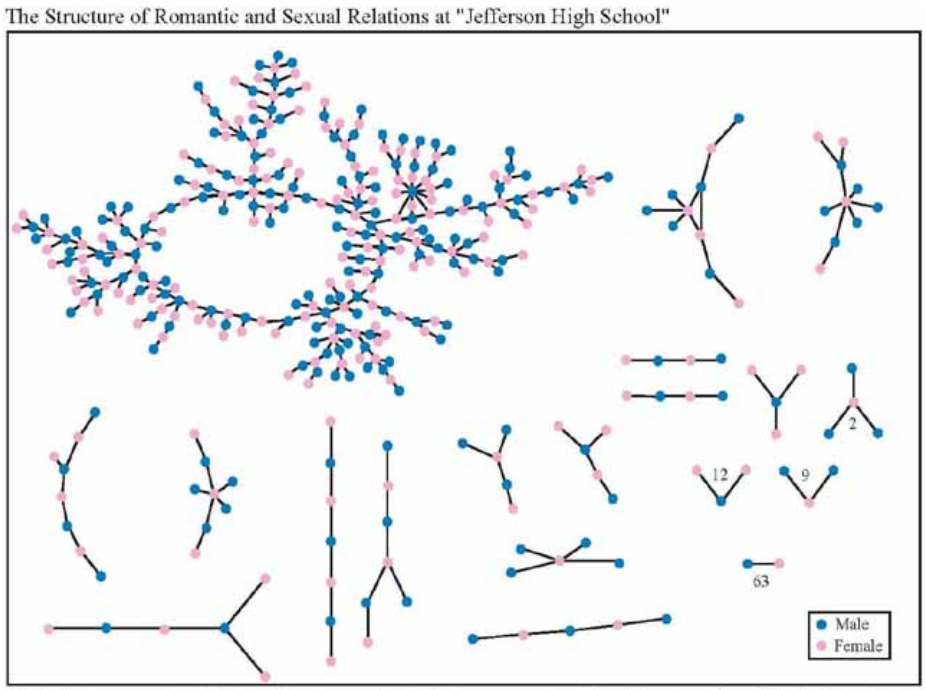

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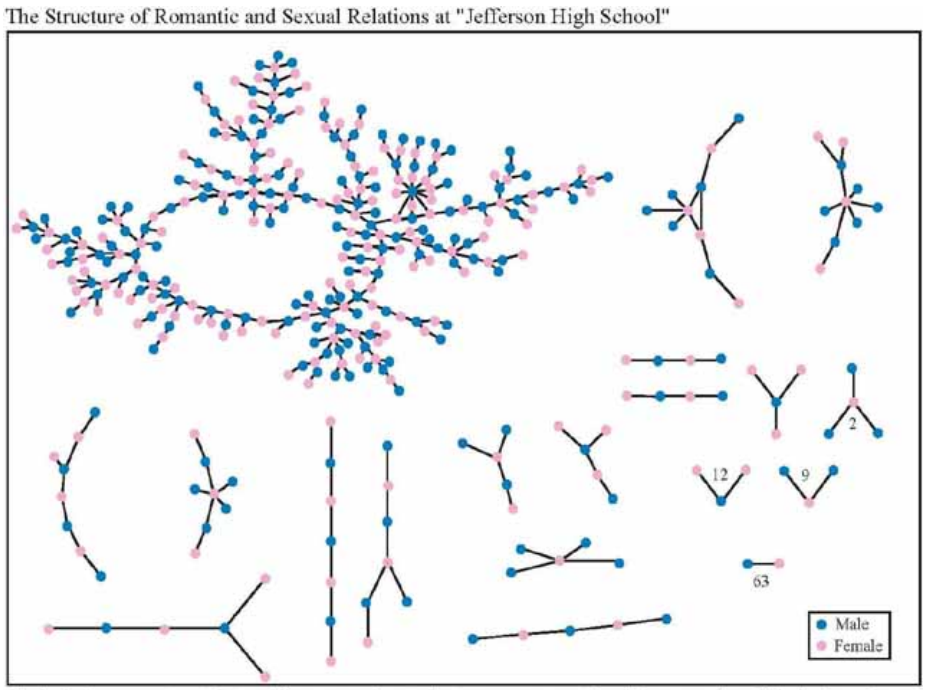

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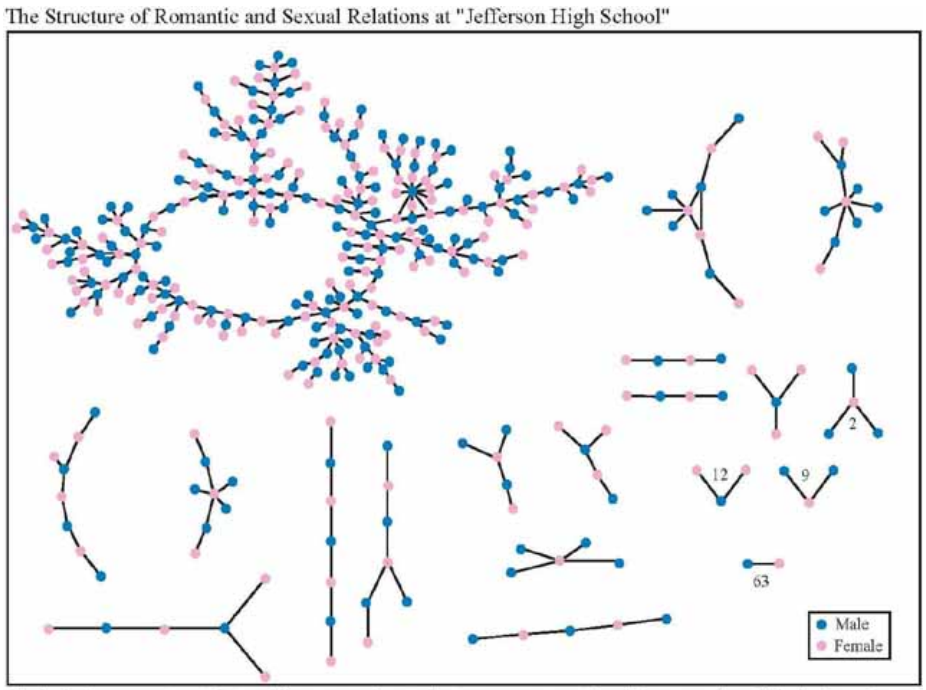

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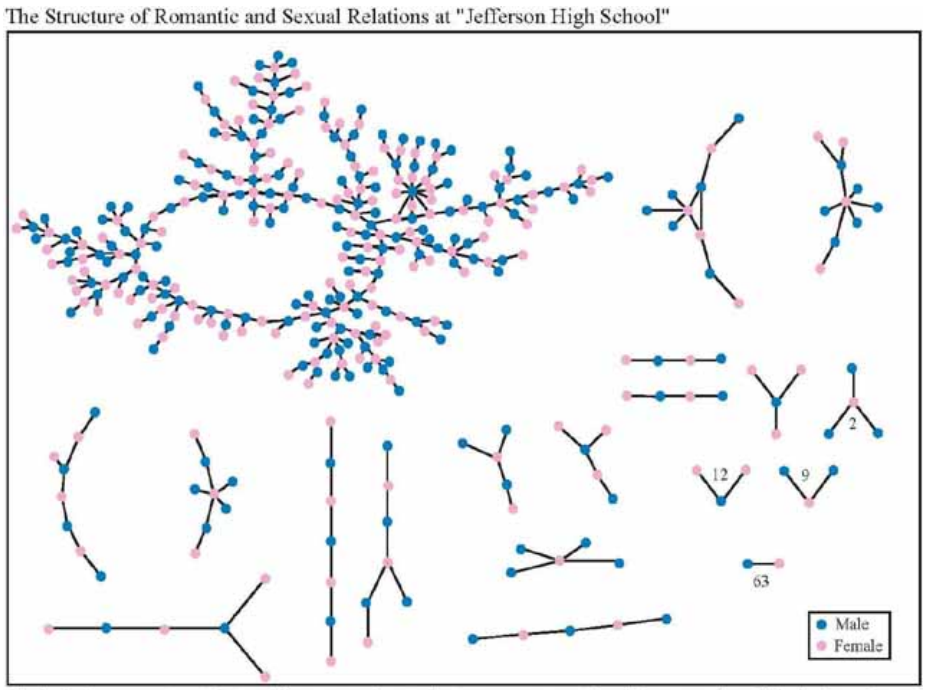

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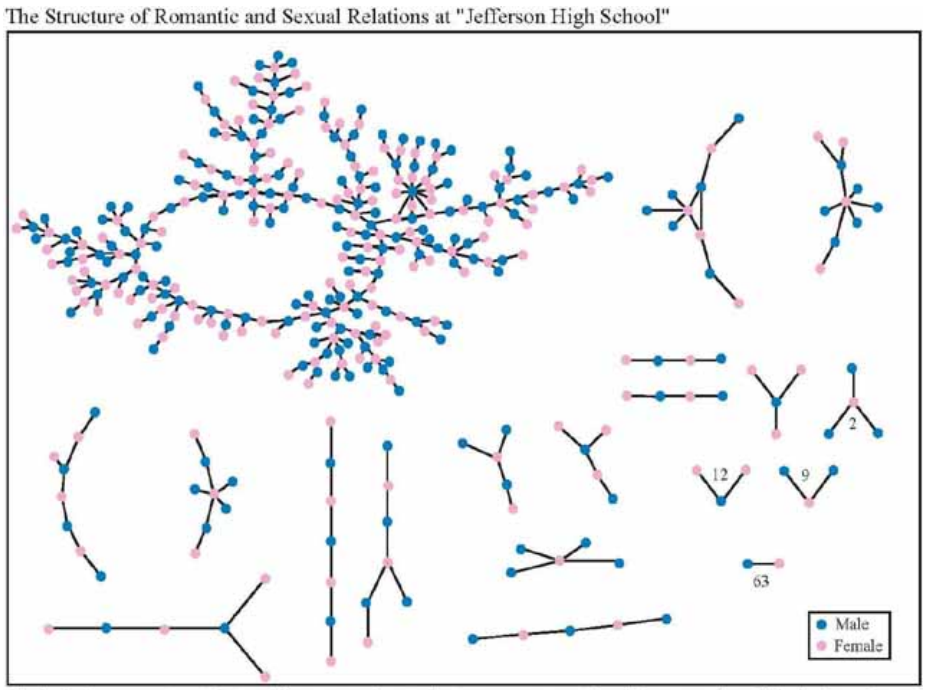

Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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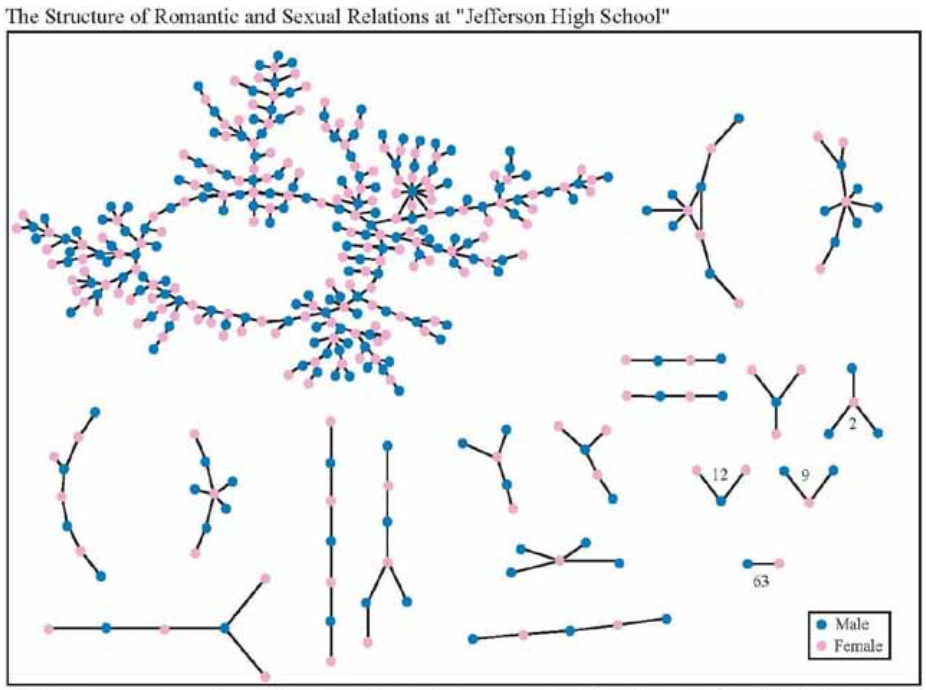

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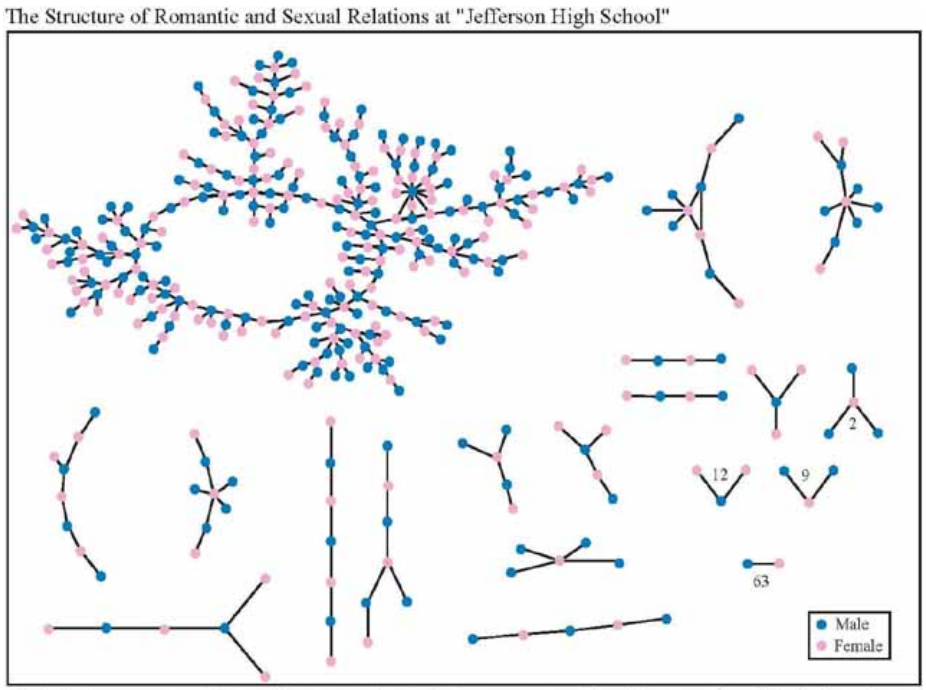

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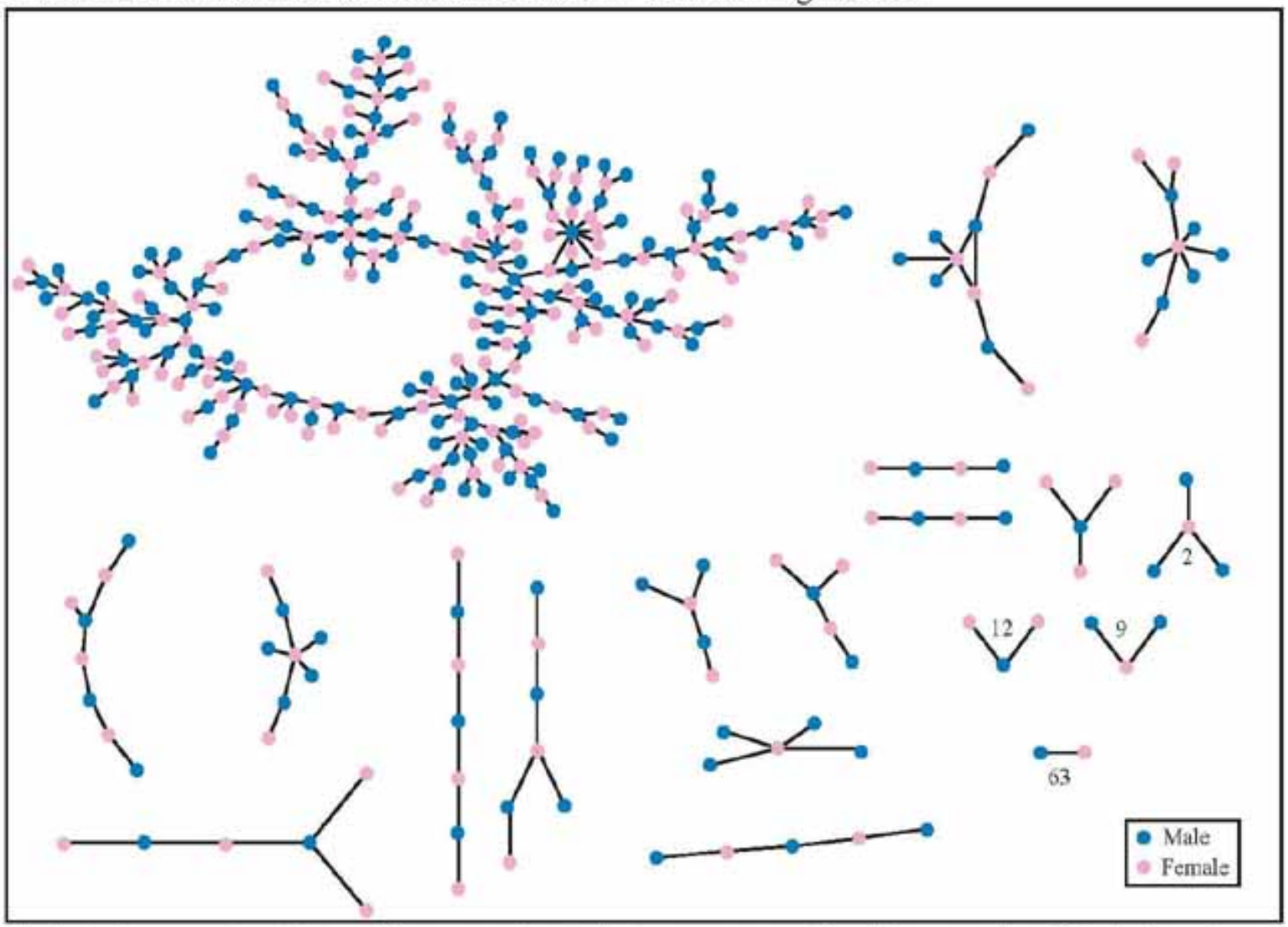
The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

Examples

Relational networks

- Consumer purchases

- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: del.icio.us (⊞), flickr (⊞)

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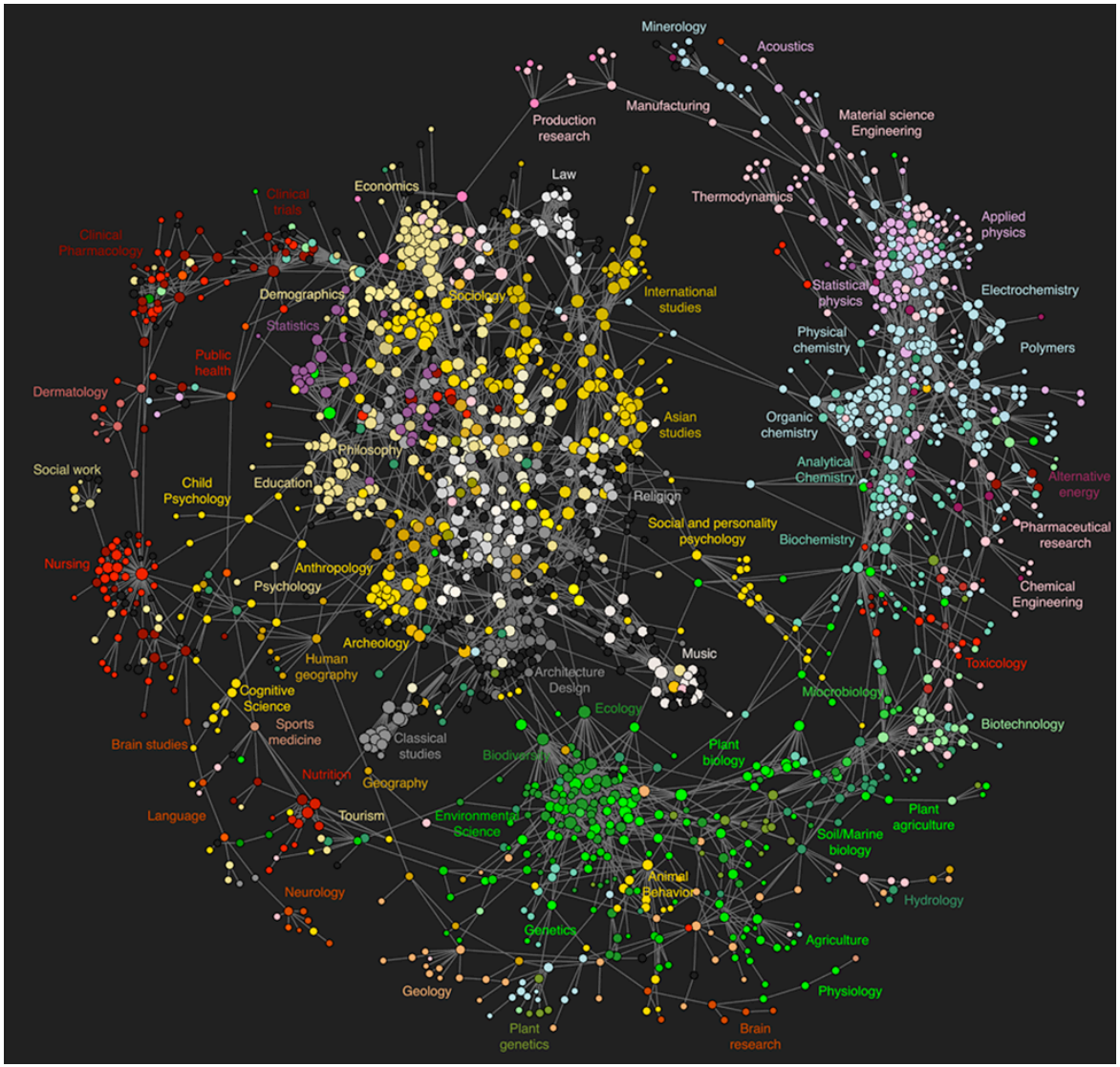
common tags cloud | list

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

Clickworthy Science:



Bollen et al. [5]

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.
- And even when renderings somehow look good:
- We need to extract digestible, meaningful aspects.

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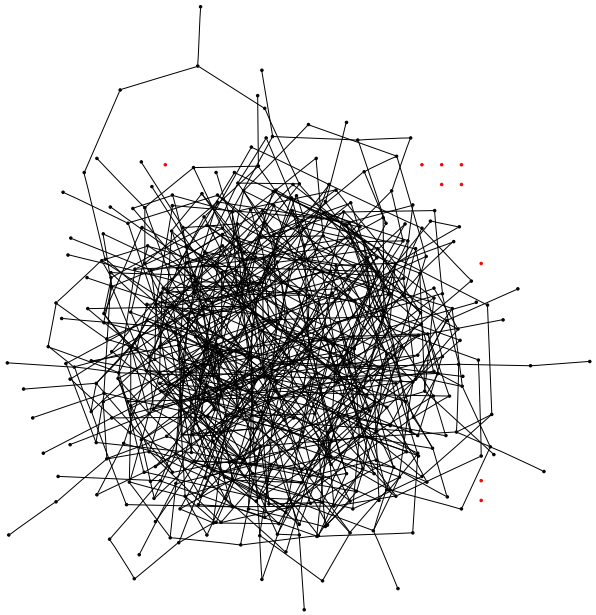

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- number of nodes $N = 500$
- number of edges $m = 1000$
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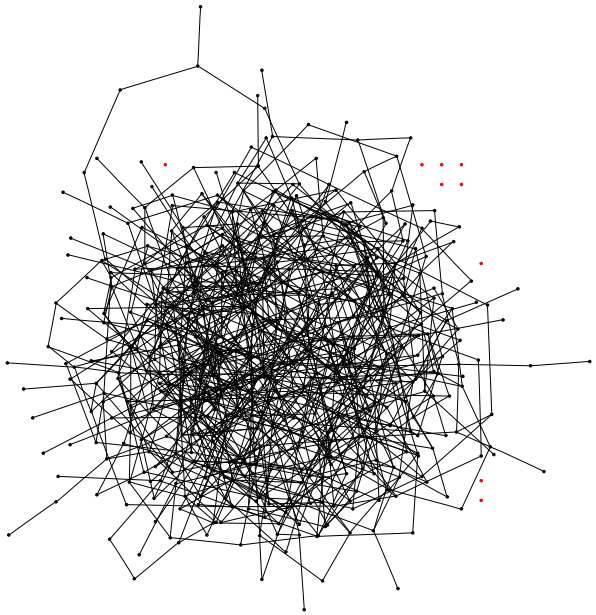

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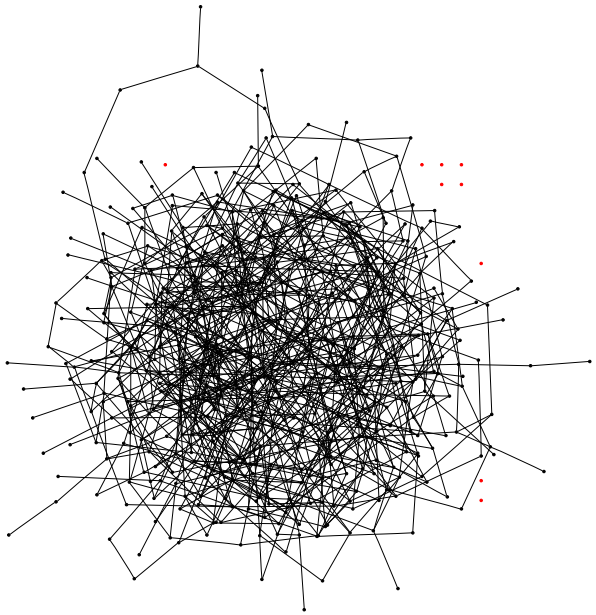

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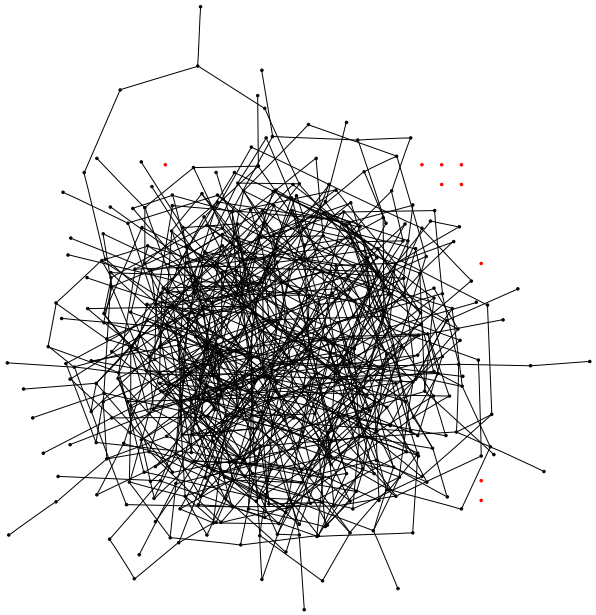

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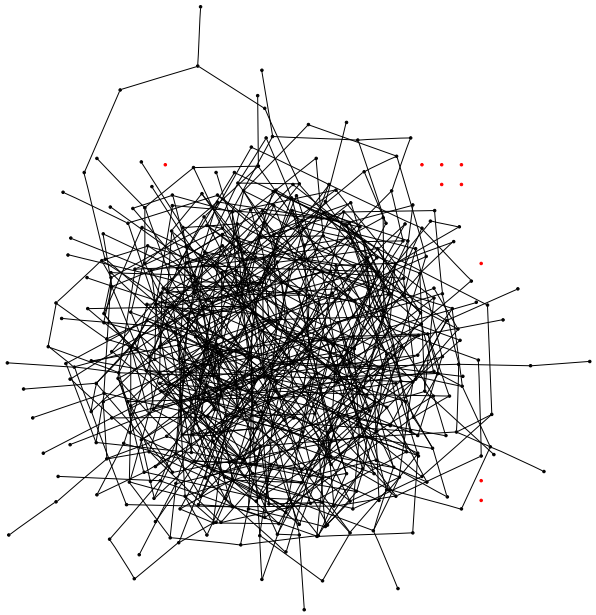

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Properties

Some key features of real complex networks:

- Degree distribution
- Assortativity
- Homophily
- Clustering
- Motifs
- Modularity
- Concurrency
- Hierarchical scaling
- Network distances
- Centrality
- Efficiency
- Robustness

- Coevolution of network structure and processes on networks.

Properties

1. Degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- Big deal: Form of P_k key to network's behavior
- ex 1: Erdős-Rényi random networks have a Poisson distribution:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
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2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[14] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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  - *Often techological or biological: Internet, protein interactions, neural networks, food webs.*

Properties

4. Clustering:

- Your friends tend to know each other.
- Two measures:

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i \text{ due to Watts \& Strogatz [21]}$$

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}} \text{ due to Newman [15]}$$

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- Small, recurring functional subnetworks
- e.g., Feed Forward Loop:

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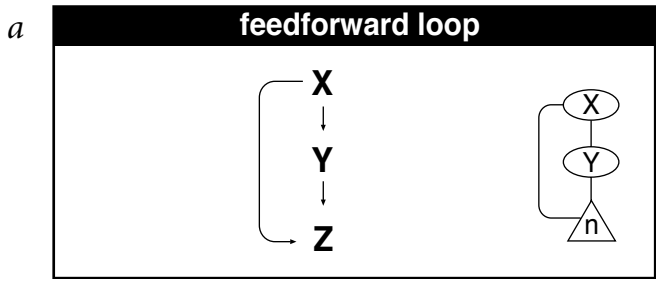

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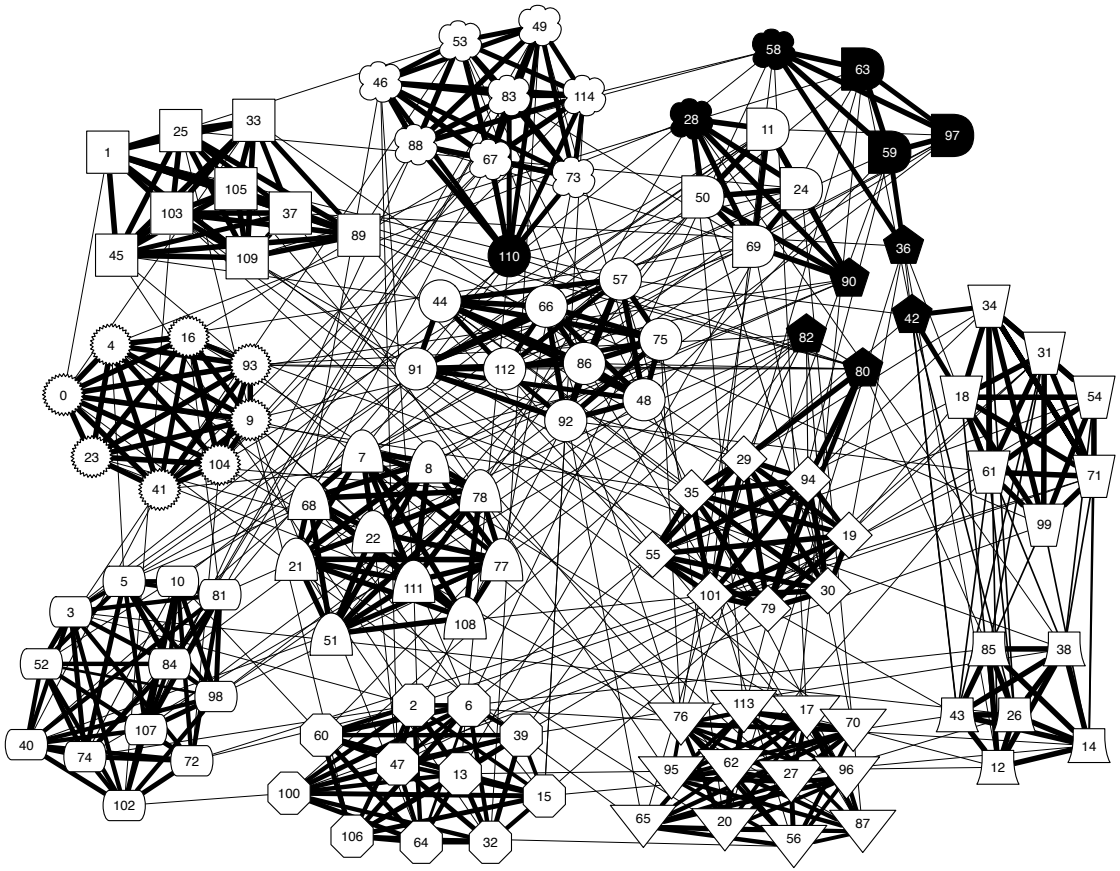

Clauset *et al.*, 2006 [7]: NCAA football

Properties

7. Concurrency:

- Transmission of a contagious element only occurs during contact [13]
- Rather obvious but easily missed in a simple model
- Dynamic property—static networks are not enough
- Knowledge of previous contacts crucial
- Beware cumulated network data!

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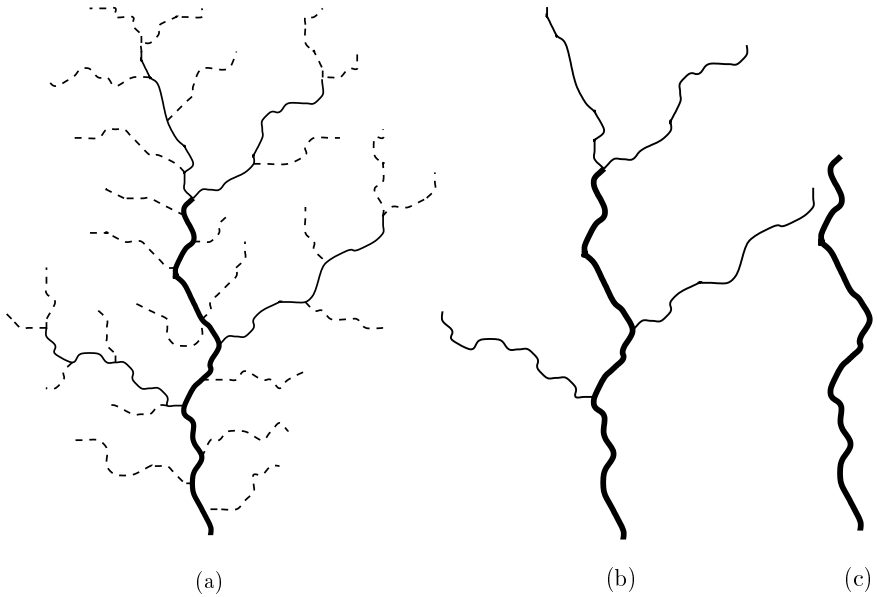
8. Horton-Strahler stream ordering:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Reveals fractal nature of natural branching networks
  - Hierarchy is not pure but mixed (Tokunaga). [18, 8]
  - Major examples: rivers and blood networks.



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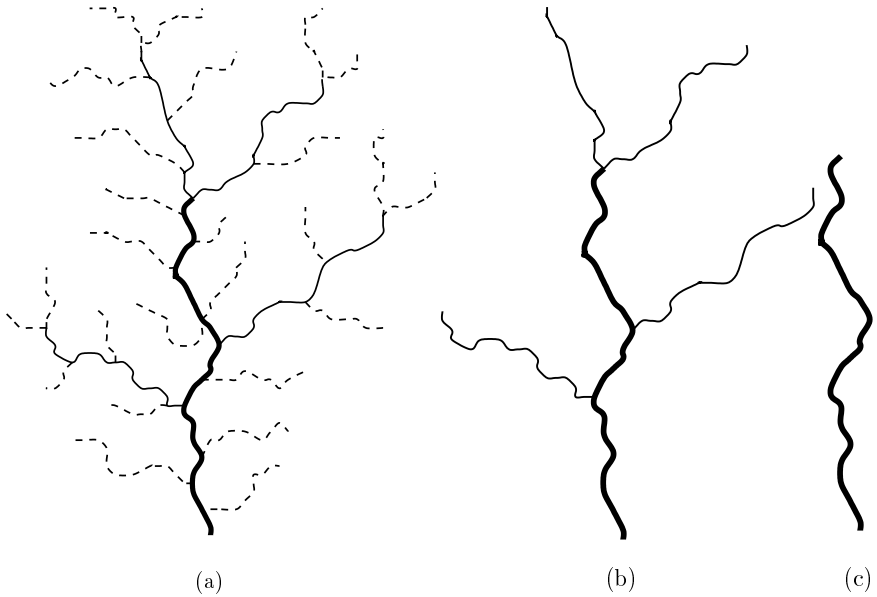

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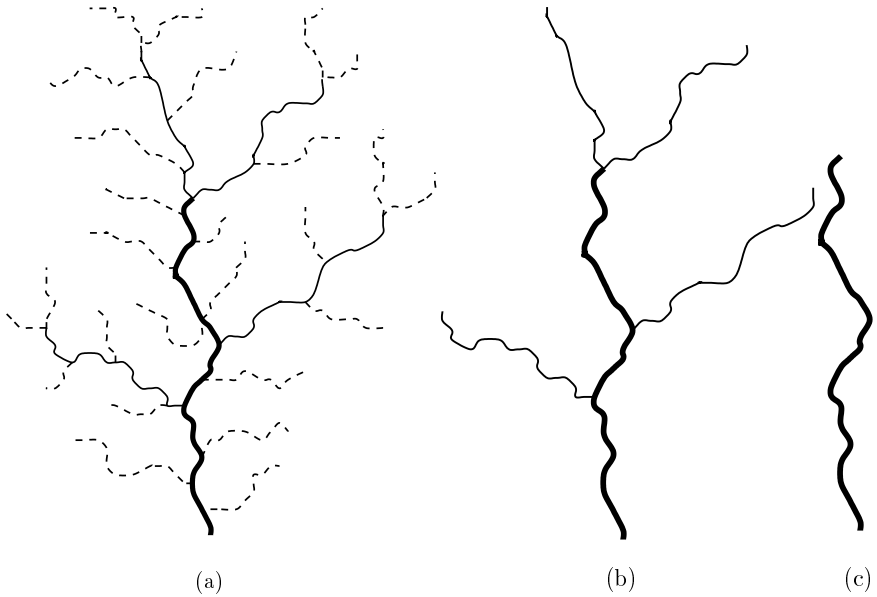

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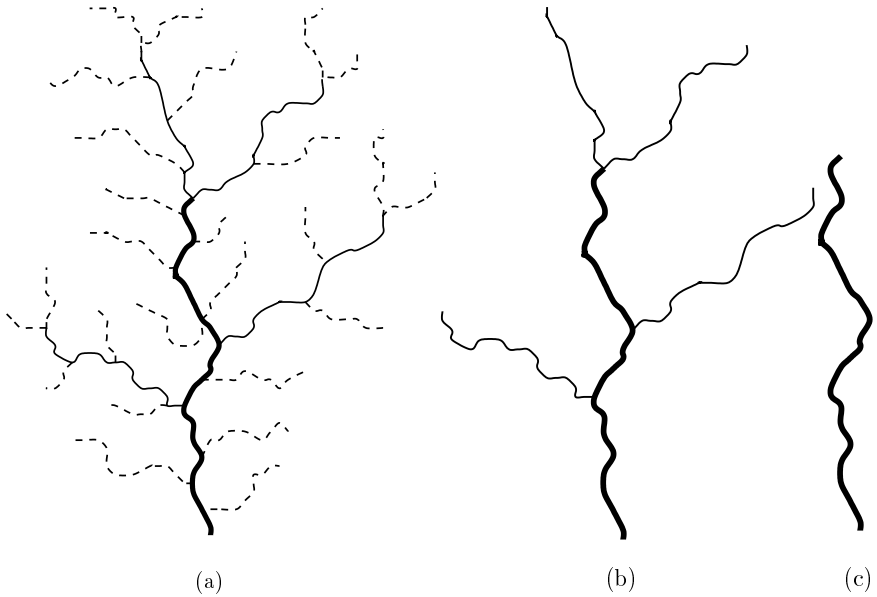

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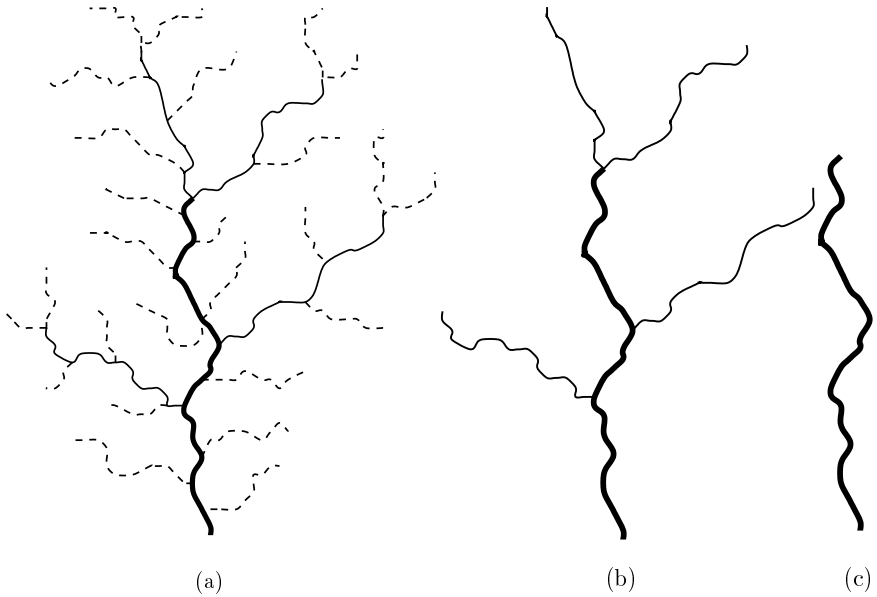

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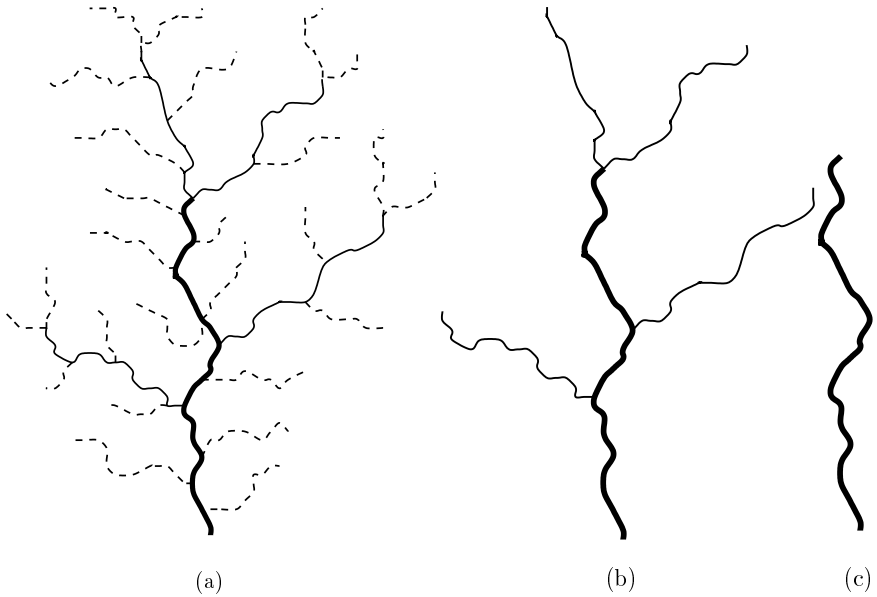

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(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
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(c) Network diameter $d_{\max}$:

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- Explosion of papers and interest since 1998/99.
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- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
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References I

[1] R. Albert and A.-L. Barabási.
Statistical mechanics of complex networks.
Rev. Mod. Phys., 74:47–97, 2002. pdf (⊞)

[2] P. W. Anderson.
More is different.
Science, 177(4047):393–396, August 1972. pdf (⊞)

[3] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf (⊞)

[4] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang.
Complex networks: Structure and dynamics.
Physics Reports, 424:175–308, 2006. pdf (⊞)

References II

[5] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
Clickstream data yields high-resolution maps of science.
PLoS ONE, 4:e4803, 2009. pdf (⊞)

[6] S. Bornholdt and H. G. Schuster, editors.
Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.

[7] A. Clauset, C. Moore, and M. E. J. Newman.
Structural inference of hierarchies in networks, 2006.
pdf (⊞)

[8] P. S. Dodds and D. H. Rothman.
Unified view of scaling laws for river networks.
Physical Review E, 59(5):4865–4877, 1999. pdf (⊞)

References III


[9] S. N. Dorogovtsev and J. F. F. Mendes.
Evolution of Networks.
Oxford University Press, Oxford, UK, 2003.


[10] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.


[11] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
IEEE Intelligent Systems, 24:8–12, 2009. pdf (⊞)


[12] J. M. Kleinberg.
Authoritative sources in a hyperlinked environment.
Proc. 9th ACM-SIAM Symposium on Discrete Algorithms, 1998. pdf (⊞)

References IV


[13] M. Kretzschmar and M. Morris.
Measures of concurrency in networks and the spread of infectious disease.
Math. Biosci., 133:165–95, 1996. pdf (⊞)


[14] M. Newman.
Assortative mixing in networks.
Phys. Rev. Lett., 89:208701, 2002. pdf (⊞)


[15] M. E. J. Newman.
The structure and function of complex networks.
SIAM Review, 45(2):167–256, 2003. pdf (⊞)


[16] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambrigde, UK, 1997.

References V


[17] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon.
Network motifs in the transcriptional regulation network of *Escherichia coli*.
Nature Genetics, pages 64–68, 2002. pdf (⊞)


[18] E. Tokunaga.
The composition of drainage network in Toyohira River Basin and the valuation of Horton's first law.
Geophysical Bulletin of Hokkaido University, 15:1–19, 1966.


[19] F. Vega-Redondo.
Complex Social Networks.
Cambridge University Press, 2007.


[20] D. J. Watts.
Six Degrees.
Norton, New York, 2003.

References VI

[21] D. J. Watts and S. J. Strogatz.
Collective dynamics of ‘small-world’ networks.
Nature, 393:440–442, 1998. pdf (⊞)