Overview of Complex Networks

Complex Networks, SFI Summer School, June, 2010

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Overview

Plan

Basic definitions

Popularity

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Complex Networks

Properties of
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Nutshell

References

Outline

Something of a plan:

- Lecture 1: Overview; Background
- Lecture 2: Random, Scale-free, and Small-World networks
- Lecture 3: Models of Contagion
- Lecture 4: Transportation networks; Discovering structure

Exciting details regarding these slides:

- ▶ Three versions (all in pdf):
    1. Presentation,
    2. Flat Presentation,
    3. Handout (2x2).
- ▶ Presentation versions are navigable and hyperlinks are clickable.
- ▶ Web links look like this (⊞).
- ▶ References in slides link to full citation at end. [2]
- ▶ Citations contain links to papers in pdf (if available).
- ▶ 50 hours of lectures → 5 hours.
- ▶ Brought to you by a concoction of LaTeX, Beamer, perl, and madness.

Bonus materials:

Graduate Course Websites:

- ▶ SFI Summer School Course (this one!):

  http://www.uvm.edu/~pdodds/teaching/courses/2010-06SFI-networks/ (⊞)

- ▶ Principles of Complex Systems (⊞), University of Vermont

- ▶ Complex Networks (⊞), University of Vermont

Textbooks:

- ▶ Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)

- ▶ David Easley and Jon Kleinberg (Economics and
  Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a
  Highly Connected World" (⊞)

Bonus materials:

Review articles:

- ▶ S. Boccaletti et al.
  "Complex networks: structure and dynamics" [4]
  Times cited: 1,028 (as of June 7, 2010)

- ▶ M. Newman
  "The structure and function of complex networks" [15]
  Times cited: 2,559 (as of June 7, 2010)

- ▶ R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)

Basic definitions

Complex System—Some ingredients:

- ▶ Distributed system of many interrelated parts
- ▶ No centralized control
- ▶ Nonlinear relationships
- ▶ Existence of feedback loops
- ▶ Complex systems are open (out of equilibrium)
- ▶ Presence of Memory
- ▶ Modular (nested)/multiscale structure
- ▶ Opaque boundaries
- ▶ Emergence—'More is Different'[2]

Basic definitions

Complex: (Latin = with + fold/weave (com + plex))
Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent
etymological investigation: (⊞)

- ▶ Opus reticulatum:
- ▶ A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]
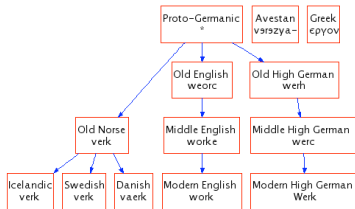
Ancestry:

Net and Work are venerable old words:

- ▶ 'Net' first used to mean spider web (King Ælfréd, 888).
- ▶ 'Work' appears to have long meant purposeful action.
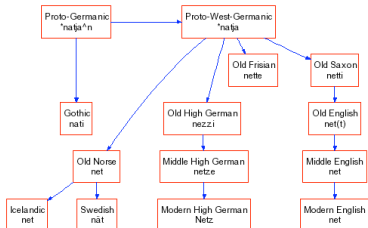
The network of Germanic 'net' words

The network of 'work' words

- ▶ 'Network' = something built based on the idea of natural, flexible lattice or web.
- ▶ c.f., ironwork, stonework, fretwork.

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- ▶ 1658–: reticulate structures in animals
- ▶ 1839–: rivers and canals
- ▶ 1869–: railways
- ▶ 1883–: distribution network of electrical cables
- ▶ 1914–: wireless broadcasting networks

- ▶ Natural → man-made
- ▶ Physical connections → Wire-less connections → abstract connections

Key Observation:

- ▶ Many complex systems
  can be viewed as complex networks
  of physical or abstract interactions.
- ▶ Opens door to mathematical and numerical analysis.
- ▶ Dominant approach of last decade of a
  theoretical-physics/stat-mechish flavor.
- ▶ Mindboggling amount of work published on complex
  networks since 1998...
- ▶ ... largely due to your typical theoretical physicist:

- ▶ *Piranha physicus*
- ▶ Hunt in packs.
- ▶ Feast on new and interesting ideas
  (see chaos, cellular automata, ...)

Popularity (according to ISI Web of Knowledge)

"Collective dynamics of 'small-world' networks" [21]

- ▶ Watts and Strogatz
  Nature, 1998
- ▶ Cited $\approx$ 4325 times (as of June 7, 2010)
- ▶ Over 1100 citations in 2008.

"Emergence of scaling in random networks" [3]

- ▶ Barabási and Albert
  Science, 1999
- ▶ Cited $\approx$ 4769 times (as of June 7, 2010)
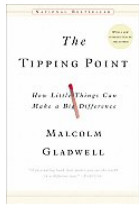- ▶ Over 1100 citations in 2008.

Popularity according to books:

The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [10]
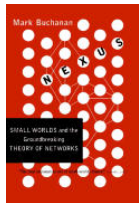
Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

Popularity according to books:
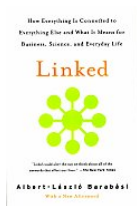
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Linked: How Everything Is Connected to
Everything Else and What It
Means—Albert-Laszlo Barabási

Six Degrees: The Science of a Connected
Age—Duncan Watts [20]

Numerous others:

- ▶ Complex Social Networks—F. Vega-Redondo [19]
- ▶ Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [16]
- ▶ Random Graph Dynamics—R. Durette
- ▶ Scale-Free Networks—Guido Caldarelli
- ▶ Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- ▶ Complex Graphs and Networks—Fan Chung
- ▶ Social Network Analysis—Stanley Wasserman and Kathleen Faust
- ▶ Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [6]
- ▶ Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [9]

More observations

- ▶ But surely networks aren't new...
- ▶ Graph theory is well established...
- ▶ Study of social networks started in the 1930's...
- ▶ So why all this 'new' research on networks?
- ▶ Answer: Oodles of Easily Accessible Data.
- ▶ We can now inform (alas) our theories
  with a much more measurable reality.*
- ▶ A worthy goal: establish mechanistic explanations.

  If this is upsetting, maybe string theory is for you...

More observations

▶ Web-scale data sets can be overly exciting.

Witness:

▶ The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
▶ "The Unreasonable Effectiveness of Data," Halevy et al. [11].

But:

▶ For scientists, description is only part of the battle.
▶ We still need to understand.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

▶ e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

▶ Links may be directed or undirected.
▶ Links may be binary or weighted.

Other spiffing words: vertices and edges.

Super Basic definitions

Node degree = Number of links per node

- ► Notation: Node i's degree = k_i.
- ► $k_i = 0, 1, 2, \ldots$
- ► Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- ► Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- ► Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

Super Basic definitions

Adjacency matrix:

▶ We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).

▶ e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

▶ (n.b., for numerical work, we always use sparse matrices.)

Examples

So what passes for a complex network?

▶ Complex networks are large (in node number)
▶ Complex networks are sparse (low edge to node ratio)
▶ Complex networks are usually dynamic and evolving
▶ Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks

- The Internet
- Road networks
- Power grids
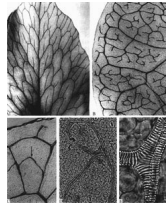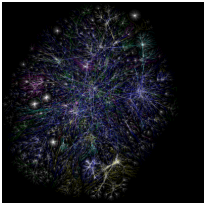
- Distribution (branching) vs. redistribution (cyclical)

Examples

Interaction networks

- ▶ The Blogosphere
- ▶ Biochemical networks
- ▶ Gene-protein networks
- ▶ Food webs: who eats whom
- ▶ The World Wide Web (?)
- ▶ Airline networks
- ▶ Call networks (AT&T)
- ▶ The Media
- ▶ Paper citations

datamining.typepad.com (⊞)

Examples

Interaction networks:
social networks

- ► Snogging
- ► Friendships
- ► Acquaintances
- ► Boards and
  directors
- ► Organizations
- ► facebook.com (⊞),
  twitter.com (⊞)

The Structure of Romantic and Sexual Relations at "Jefferson High School"
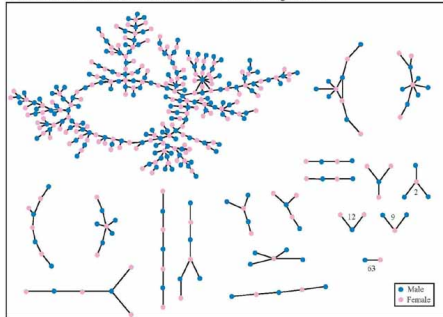
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- ► 'Remotely sensed' by: email activity, instant
  messaging, phone logs (*cough*).

Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"
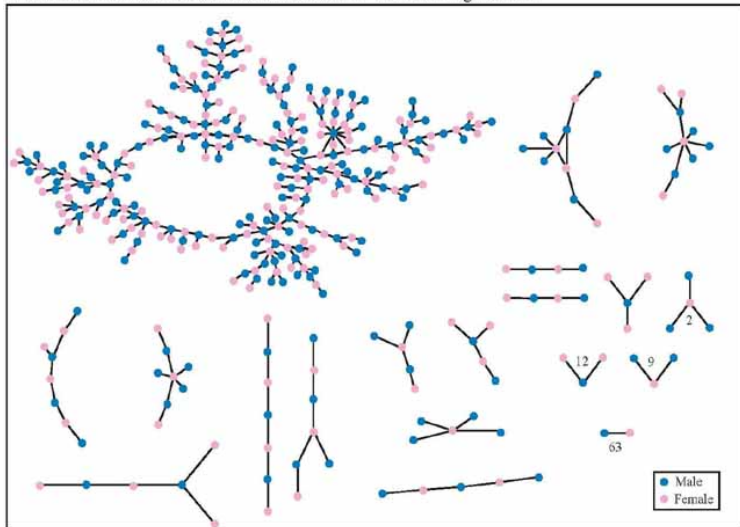
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

Examples

Relational networks

- Consumer purchases
  (Wal-Mart: ≈ 1 petabyte $= 10^{15}$ bytes)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: del.icio.us (⊞), flickr (⊞)

common tags   cloud | list

community daily dictionary education encyclopedia
english free imported info information internet knowledge
learning news reference research resource
resources search tools useful web web2.0 wiki
wikipedia

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Clickworthy Science:

Bollen et al. [5]

A notable feature of large-scale networks:

► Graphical renderings are often just a big mess.
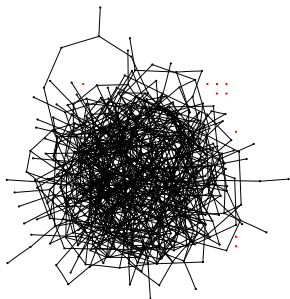
⇐ Typical hairball

► number of nodes $N = 500$

► number of edges $m = 1000$

► average degree $\langle k \rangle$ = ?4

► And even when renderings somehow look good:
"That is a very graphic analogy which aids
understanding wonderfully while being, strictly
speaking, wrong in every possible way"
said Ponder [Stibbons] —*Making Money*, T. Pratchett.

► We need to extract digestible, meaningful aspects.

Properties

Some key features of real complex networks:

- Degree distribution
- Assortativity
- Homophily
- Clustering
- Motifs
- Modularity

- Concurrency
- Hierarchical scaling
- Network distances
- Centrality
- Efficiency
- Robustness

- Coevolution of network structure and processes on networks.

Properties

1. Degree distribution P_k

  ▶ P_k is the probability that a randomly selected node has degree k
  ▶ Big deal: Form of P_k key to network's behavior
  ▶ ex 1: Erdős-Rényi random networks have a Poisson distribution:
  $$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$
  ▶ ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
  ▶ We'll come back to this business soon...

Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network: [14] similar degree nodes connecting to each other.
  - *Often social: company directors, coauthors, actors.*
- Disassortative network: high degree nodes connecting to low degree nodes.
  - *Often techological or biological: Internet, protein interactions, neural networks, food webs.*

Properties

4. Clustering:

- Your friends tend to know each other.
- Two measures:

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i \text{ due to Watts \& Strogatz}^{[21]}$$

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}} \text{ due to Newman}^{[15]}$$

- C_1 is the average fraction of pairs of neighbors who are connected.
- Interpret C_2 as probability two of a node's friends know each other.

Properties

5. Motifs:

► Small, recurring functional subnetworks

► e.g., Feed Forward Loop:
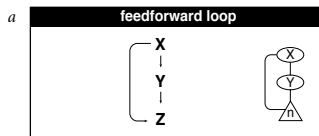
Shen-Orr, Uri Alon, *et al.*[17]

Properties

6. modularity:
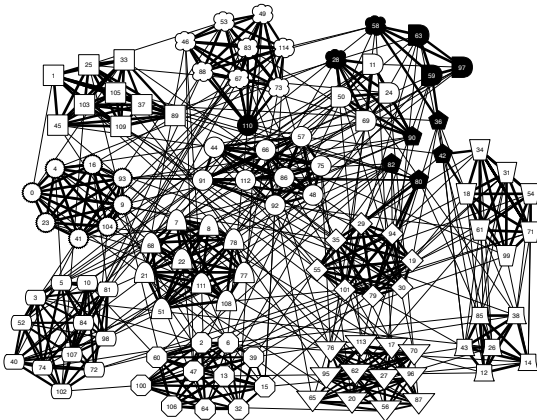
Clauset *et al.*, 2006 [7]: NCAA football

Properties

7. Concurrency:

- ▶ Transmission of a contagious element only occurs during contact [13]
- ▶ Rather obvious but easily missed in a simple model
- ▶ Dynamic property—static networks are not enough
- ▶ Knowledge of previous contacts crucial
- ▶ Beware cumulated network data!

Properties

8. Horton-Strahler stream ordering:

- ▶ Metrics for branching networks:
    - ▶ Method for ordering streams hierarchically
    - ▶ Reveals fractal nature of natural branching networks
    - ▶ Hierarchy is not pure but mixed (Tokunaga). [18, 8]
    - ▶ Major examples: rivers and blood networks.

(a)   (b)   (c)
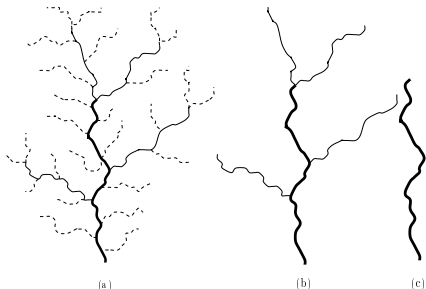
- ▶ Beautifully described but poorly explained.

Properties

9. Network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
- Weighted links can be accommodated.

Properties

9. Network distances:

(c) Network diameter $d_{\max}$:

- Maximum shortest path length in network.

(d) Closeness $d_{\mathrm{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

- Average 'distance' between any two nodes.
- Closeness handles disconnected networks ($d_{ij} = \infty$)
- $d_{\mathrm{cl}} = \infty$ only when all nodes are isolated.

Properties

10. Centrality:

- Many such measures of a node's 'importance.'
- ex 1: Degree centrality: k_i.
- ex 2: Node i's betweenness
  = fraction of shortest paths that pass through i.
- ex 3: Edge ℓ's betweenness
  = fraction of shortest paths that travel along ℓ.
- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [12])

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

Nutshell:

Overview Key Points (cont.):

- ▶ Obvious connections with the vast extant field of graph theory.
- ▶ But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
- ▶ Two main areas of focus:
    1. Description: Characterizing very large networks
    2. Explanation: Micro story $\Rightarrow$ Macro features
- ▶ Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure,...
- ▶ Still much work to be done, especially with respect to dynamics...

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